

Confinement and $U(1,3)$ symmetry of color particles in complex phase space

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Abstract

It is shown that a universal confining potential for hadron constituents can be obtained with the help of $U(1,3)$ symmetry in a complex phase space. Parameters of this potential are determined on the basis of spectroscopic data for hadrons and results of lattice QCD calculations. We argue that the account of the $U(1,3)$ symmetry is important for a description of strong interactions of quarks and gluons in a nonperturbative QCD domain at large interaction distances.

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1. Introduction

Color particles such as quarks and gluons are observed in indirect measurements in the hadron physics, but they have never been seen as usual particles in asymptotic free states. Although perturbative QCD calculations of processes at large momentum transfers are confirmed by experimental data, rigorous calculations of many nonperturbative effects are impossible at present even in the framework of the lattice QCD (lQCD). The confinement of color particles is one amongst the main phenomena of the nonperturbative QCD (nQCD) and a good deal of effort are demanded for incorporating it into a complete theory of strong interactions [1]. In spite of the fact that the confinement, perhaps, will be validated with the QCD Lagrangian, this property of QCD can be connected with some symmetry, which has not been taking into account yet. For instance, it may be a kinematical symmetry connected with generalized space-time properties of quarks and gluons. It is well known that the Poincaré symmetry, which is the space-time symmetry of the orthodox relativistic quantum field theory, is originated from the isotropy and the homogeneity of the Minkowski space-time and is based upon observations of macro- and micro-phenomena concerning ordinary physical bodies and particles.

In the present paper we suppose that the confinement of color particles can be understood as a corollary of a generalized symmetry, which possess non-perturbative interactions of color particles. We consider the $U(1,3)$ group as the group of symmetry of such kind. It is possible, that the $U(1,3)$ symmetry belong to a class of approximated or exact nQCD symmetries. In any case the $U(1,3)$ symmetry in a complex phase space allows easily to include the confinement of color particles into consideration and in the framework of potential models give an explicit form of a confining potential.

The $U(1,3)$ symmetry has been used in physics for the first time as the group of a dynamic symmetry of a isotropic oscillator [2]. Its discrete unitary representations were applied for a description of hadron properties [3, 4]. In Ref. [5] the $U(1,3)$ symmetry of a complexified theory of gravitation has been

found. This symmetry has been proposed in Refs. [6] as a extended symmetry in a complex phase space for quarks and gluons. $SU(1,3)$ symmetrical coherent states were studied as well [7] and a generalized $U(1,3)$ symmetrical quantum mechanics was considered in Refs. [8].

The paper is organized as follows: sections 2 and 3 are devoted to a brief description of $U(1,3)$ transformations and a realization of their representations on generalized quark fields. An $U(1,3)$ symmetrical confining potential and possible values of its parameters are given in section 4. Conclusions are found in section 5.

2. $U(1,3)$ symmetry

The pseudounitary $U(1,3)$ group can be defined as a group of transformations in a complex phase space C_4 with vectors $c_\mu = q_\mu - i\kappa^{-1}p_\mu$, $\mu = 0, 1, 2, 3$, which leave invariant the following Hermitian form:

$$|c|^2 = c_\nu c_\mu^* \eta^{\mu\nu} = c_0 c_0^* - c_1 c_1^* - c_2 c_2^* - c_3 c_3^*, \quad (1)$$

where $*$ is a complex conjugation, $\eta^{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$, $\eta^{\mu\nu}\eta_{\nu\sigma} = \delta_\sigma^\mu$, κ is a constant with dimensions of $[M^2]$ in the natural system of units with $\hbar = c = 1$. It is convenient to define contravariant vectors: $c^\mu = \eta^{\mu\nu} c_\nu^*$, then the invariant Hermitian form (1) can be written as $c_\mu c^\mu$. Translations in the complex space C_4 are denoted through $m_\mu = p_\mu + i\kappa q_\mu$.

Let us consider along with the complex phase space C_4 a complex Grassman algebra G_4 with generating elements η^α , $\alpha = 0, 1, 2, 3$ and a set of functions F_i , which are defined on $C_4 \otimes G_4$ and have the following form:

$$F_i(c^\mu, c_\mu; \eta^\alpha) = f_i(c^\mu, c_\mu) + \sum_{k>0} \sum_{\alpha_1 < \dots < \alpha_k} f_{i\alpha_1 \dots \alpha_k}(c^\mu, c_\mu) \times \eta^{\alpha_1} \dots \eta^{\alpha_k} \quad (2)$$

One can choose a subset F_{iD} , consisting of those functions F_i , which satisfy an $U(1,3)$ invariant equation of the Dirac type [6]:

$$(i\eta^\mu \frac{\partial}{\partial c^\mu} + i \frac{\partial}{\partial \eta^\mu} \frac{\partial}{\partial c_\mu} - \frac{C}{2}) F(c^\mu, c_\mu; \eta^\alpha) = 0 \quad (3)$$

In the quantum case, when q_μ and p_μ are noncommutative quantities, one should consider functions only in a coordinate representation or in a momentum representation. Thus we perform a substitution from c^μ and c_μ variables to q_μ and p_μ variables and restrict ourselves, for instance, only those functions, which depend on coordinates and transform under irreducible representation (IR) of the $U(1, 3)$ group. The following relations are useful, when one make this substitution:

$$\frac{\partial}{\partial c^\mu} = \frac{1}{2} \left(\frac{\partial}{\partial q^\mu} - i\kappa \frac{\partial}{\partial p^\mu} \right), \quad \frac{\partial}{\partial c_\mu} = \frac{1}{2} \left(\frac{\partial}{\partial q_\mu} + i\kappa \frac{\partial}{\partial p_\mu} \right).$$

$$\gamma_\mu = \eta_\mu + \frac{\partial}{\partial \eta^\mu}, \quad \gamma'_\mu = i(\eta_\mu - \frac{\partial}{\partial \eta^\mu}), \quad (4)$$

where $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$, $\{\gamma_\mu, \gamma'_\nu\} = 0$.

3. Realization of the $U(1, 3)$ symmetry on generalized quark fields

If we take into account that any quark belong some hadron, thus quarks (or other color particles) should be described with bilocal fields $\Psi_i(q_\mu, Q_\mu)$, where q_μ are quark coordinates and Q_μ are hadron ones. In order to satisfy the translation invariance condition the fields $\Psi_i(q_\mu, Q_\mu)$ are reduced to fields Ψ'_i , which depend on differences $q_\mu - Q_\mu$. Analogously, an interaction potential, which one can add in the equation (3), should be dependent on the $q_\mu - Q_\mu$. In the following we restrict ourselves by a consideration of a four-dimensional fundamental $U(1, 3)$ IR, which is a bispinor representation of the Lorentz group as well. Then for a generalized quark field $\psi(q_\mu - Q_\mu)$, using (3), (4) and (5), one can write the Dirac equation in the following form:

$$(i\gamma^\mu \frac{\partial}{\partial (q - Q)^\mu} - \gamma^\mu V_\mu(q_\mu - Q_\mu) - S(q_\mu - Q_\mu) - m)\psi(q_\mu - Q_\mu) = 0, \quad (5)$$

where $V_\mu(q_\mu - Q_\mu)$ is a Lorentz vector part of an interaction potential, while $S(q_\mu - Q_\mu)$ is a Lorentz scalar part.

4. $U(1, 3)$ symmetrical confining potential

For a $U(1, 3)$ invariant theory in C_4 a Poincaré mass for a quark m is coordinate dependent and

should satisfied the following equation:

$$m_C^2 = m^2(q_\mu - Q_\mu) + \kappa^2(q_\mu - Q_\mu)(q^\mu - Q^\mu) \quad (6)$$

Eq. (6) allows to define the coordinate dependence of a quark mass and gives the form of a confining potential unambiguously. Moreover, the confining potential is a scalar with respect to the Lorentz transformations. This fact is in accordance with allowable transformation properties of a confining potential in the QCD [9].

For calculations of bound state characteristics one can use a simultaneous approximation because of, as usual, steady characteristics of a bound state are needed. If N particles with coordinates $q_{i\mu}$, $i = 1, \dots, N, \mu = 0, 1, 2, 3$ interact each other and P is their total momentum, then one can impose constraints on a range of variables $q_{i\mu}$: $P^\mu q_{1\mu} = P^\mu q_{2\mu} = \dots = P^\mu Q_\mu$. Thus in the rest frame of a hadron the static $U(1, 3)$ invariant potential for each i -th quark is:

$$V_{Si}((\mathbf{q}_i - \mathbf{Q})^2) = \sqrt{m_{Ci}^2 + \kappa^2(\mathbf{q}_i - \mathbf{Q})^2} \quad (7)$$

The scalar potential (7) provides the confinement for color particles, which is linearly growing at large distances $|\mathbf{q}_i - \mathbf{Q}|$ with the coefficient equal to the parameter κ . If one consider only two interacting particles, for instance, a quark and an antiquark, then the confining potential behave as κr at large separations r between a quark and an antiquark. In the usually accepted notation $\kappa = \sigma$, where σ is the so called string tension. According to the results of Ref. [10], the numerical value of σ is equal to $0.20 \pm 0.01 \text{ GeV}^2$, however, if one include results obtained in other models [11], then the error value should be enlarged $\sigma = 0.20 \pm 0.02 \text{ GeV}^2$. If σ is known it is possible to find two constants concerning interactions in a confinement domain, according to the following relations: $m_c l_c = 1$, $\sigma = m_c / l_c$, i.e. m_c and l_c , with dimensions of the mass and the length, respectively. Taking into account the numerical value of σ written above one can obtain that the confinement mass m_c is equal to $0.45 \pm 0.02 \text{ GeV}$, while the confinement length l_c is equal to $0.44 \pm 0.02 \text{ fm}$. It is consistent with the facts that a hadron formation starts when a string

length between opposite color charges become larger or of the order of l_c and typical transversal momenta p_\perp of hadrons produced are of the order of m_c .

In the framework of potential models a vector part of an interaction potential, for instance, between a quark and an antiquark, is taken in the quasi-Coulombic form: $V(r) = -4\alpha_s/3r$, where α_s is a strong interaction constant at small interaction distances. Thus, if we take the same vector potential between a quark and an antiquark, and the sum of two scalar potentials (7) with $U(1,3)$ constants m_{C1} for a quark and m_{C2} for an antiquark, then the total static potential can be written as

$$V_{tot}(r) = -4\alpha_s/3r + C_V + \beta(C_S + \sqrt{m_{C1}^2 + (\kappa r/2)^2} + \sqrt{m_{C2}^2 + (\kappa r/2)^2}), \quad (8)$$

where β is the known Dirac matrix, C_V and C_S are two arbitrary constants for vector and scalar parts of $V_{tot}(r)$, correspondingly. Note, that the dependence of the scalar potential versus r is not strictly linear, so if we expand $V_S(r)$ at large values of r , the following expression can be obtained:

$$V_S(r) = C_S + \sigma r + \frac{m_{C1}^2 + m_{C2}^2}{\sigma r} + \dots \quad (9)$$

It is interesting to identify the third term in the expansion (9) with the Lüscher term, which is verifiable by IQCD calculations [12]. In this case the sum $m_{C1}^2 + m_{C2}^2$ is equal to $-\pi\sigma/12$.

5. Conclusions

We conclude that the confinement of color particles is easily incorporated into the framework of $U(1,3)$ invariant models with the square root scalar potentials (7). The parameters of these potentials can be estimated with the help of potential model, string model and lattice QCD results. The asymptotic confining potential is linearly growing at large distances and is invariant under the Lorentz group and a change of quark flavors. Moreover the expansion of the confining potential at large distances gives the term, which can be identified with the well known Lüscher term. These facts give the support for the importance of the account of the $U(1,3)$ symmetry in

the complex phase space C_4 for interactions of quarks and gluons at large distances in the nonperturbative QCD domain.

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